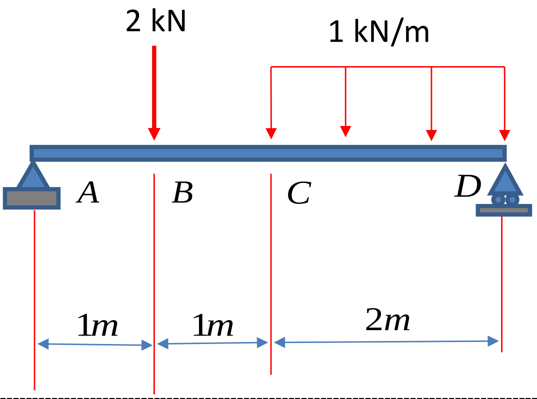
For the next structure shown at the figure below, calculate:

a) The elastic equation which provides vertical displacement at any point of the beam

b) Sag or deflection at x =2

Data:

$\mathbf{E = 210\ GPa;\ I_x = 8.10^{-8} m^4}$



a) Deflection

The first step is calculating the reaction forces:

$$\sum M_D = 0 \rightarrow 4V_a - 2.3 - 2.1.1 = 0$$

$\mathbf{V_A = 2\ kN}$

$$\sum F_y = 0 \rightarrow V_D = 2 + 1.2 - 2 = 2$$

$\mathbf{V_D = 2\ kN}$

$$\sum F_x = 0 \rightarrow \mathbf{H_A = 0\ kN}$$

Now the general equation of the elastic is to be used:

$$\mathbf{y = y_0 + \theta x + \frac{\sum M(x-a)^2}{2EI} + \frac{\sum P(x-b)^3}{6EI} + \frac{\sum q(x-c)^4}{24EI}}$$

Where:

y(x): deflection at any point

y_0 and θ are boundary conditions that need to be determined.

$\frac{\sum M(x-a)^2}{2EI}$: is the term for beding moments

$\frac{\sum P(x-b)^3}{6EI}$: is used for punctual forces

$\frac{\sum q(x-c)^4}{24EI}$: is used for distributed forces

a, b and c: distance from the origin.

At this point, we shall use the corresponding term, in function of the element found in the beam. That is why we had to previously compute the reaction forces. Using the superposition principle, it is possible to calculate the elastic equation having several elements.

Boundary conditions:

First, and since we set the reference origin at $x = 0$, we have that $y_0 = 0$. So the resulting equation is:

$$y = \theta x - \left[\frac{2}{6EI}x^3\right]_1 + \left[\frac{2.(x-1)^3}{6EI}\right]_2 + \left[\frac{1.(x-2)^4}{24EI}\right]_3$$

Now, using the other boundary condition at the other support $y(x = 4) = 0$

$$0 = 4\theta - \left[\frac{2.4^3}{6EI}\right]_1 + \left[\frac{2.(4-1)^3}{6EI}\right]_2 + \left[\frac{(4-2)^4}{24EI}\right]_3$$

$$-4\theta = \frac{1}{24EI}\left(-4.2.4^3 + 4.2.27 + 2^4\right)$$

$$\boldsymbol{\theta = \frac{280}{96EI}}$$

Finally, since we know the boundary conditions, we obtain the elastic equation of the beam:

$$\mathbf{y(x) = \frac{280}{96EI}x - \left[\frac{2}{6EI}x^3\right]_1 + \left[\frac{2.(x-1)^3}{6EI}\right]_2 + \left[\frac{(x-2)^4}{24EI}\right]_3}$$

This equation provides the vertical displacement at any point of the beam.

b) Deflection at x= 2 m.

We substitute in prior equation for x=2:

$$y(x) = \frac{1}{96EI}(280.2 - 8.16.2 + 2.16)$$

$$\mathbf{y(x=2) = 2,08.10^{-4}\ m}$$